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The measurement of wind at altitude by airborne instruments

by

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(Meteorological Office)

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

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1. Introduction

1.1. In 1955 a Marconi navigational system based on the Doppler principle was installed in a jet aircraft of the Meteorological Research Flight. This system enables the ground speed and the angle of drift between the ground speed vector and the airspeed vector to be evaluated to high accuracy by means of radio techniques. The purpose of this investigation was to determine the errors involved when this equipment is used to measure the wind.

1.2. With this equipment, a wind measurement requires the determination of four basic quantities, (a) the heading of the aircraft, (b) the true airspeed (i.e., the speed of the aircraft relative to the air which, in turn, can only be derived from the indicated airspeed, pressure altitude and true air temperature), (c) the ground speed as measured by the radio equipment and (d) the drift angle as measured by the radio equipment. The wind is the vector difference between the airspeed vector given by (a) and (b) and the ground speed vector given by (a), (c) and (d). The method used to determine the wind is given in Appendix I which also includes the calculation of errors. Each of the measured quantities is considered below.

1.2.1. Provided the heading is kept constant the error in the wind direction is equal in magnitude to the error in the heading and the error in the wind speed is a second order effect. It is, however, necessary to ensure that the heading is not varied substantially during any series of measurements and for this reason the best possible measurement of heading was always obtained. This was achieved by installing a completely separate additional gyro compass and taking photographs of its master compass presentation. This can be read to half a degree compared to the one or two degrees of the repeater compasses. Random reading errors of 0.25° in heading arising in this way can be regarded as similar errors in drift angle and will be treated as such.

1.2.2. Of all the necessary measurements, that of airspeed is the least accurate. It is subject to all the errors in the pressure indicating instruments (e.g., static and pitot position errors), instrument errors, and temperature errors including variations of the speed correction coefficient. Using the best available data and laboratory calibrations of the various instruments, both systematic and random residual errors can still be expected. The first object of the calibration was to obtain values of these residual errors which could be used to correct and interpret the observations made in subsequent flights.

1.2.3. The quoted maximum error of the ground speed indicator is 0.3 per cent and the instrument is calibrated electronically before flight to be within this tolerance.

/1.2.4.

1.2.4. The measurement of drift is susceptible to both systematic and random errors; the former arise from minor misalignments of the aerials and the latter are due to sideslip, yaw, etc. It was decided, therefore, that the second part of the calibration should be an attempt to find the systematic errors in the measurement of drift.

1.3. The most direct method of obtaining calibrations is to use simultaneous independent wind measurements and probably the best practical approach is to fire smoke puffs from the aircraft and to track the motion of the smoke visually from the ground, readings being taken in the aircraft at the same time. Unfortunately, owing to unsuitable weather conditions, this has not yet been done, but it has been possible to obtain a calibration using two other methods which are not so direct in that they assume constancy of wind in time and space over periods of up to one hour and distances up to 50 miles compared with the 5 minutes or so and distances up to 10 miles involved in the measurement by the smoke puff technique.

1.3.1. Various geometrical air tracks were flown and a theory is derived below enabling a correction factor K to be evaluated for the airspeed modulus and a systematic error γ in the angle of drift to be computed. These flights are discussed below under the heading "Geometrical Flights".

1.3.2. Seven flights were made near the time of release of a radiosonde balloon and in the region of a radiosonde station. Interpolations from the displacement winds of the radiosonde were then used to evaluate the same two constants K and γ by resolution of these winds along and at right angles to the heading of the aircraft. These flights are referred to below as "Comparison Flights".

2. List of Symbols used

Symbol	Definition	Units
α	the angle of drift	degrees
$\beta = \alpha+\gamma$	the corrected angle of drift	degrees
γ	the correction to the angle of drift	degrees
Δx	the error in the quantity x	-
θ	the heading of the aircraft	degrees true from North
ϕ	the wind direction	degrees true from North
a	the speed correction coefficient of the thermometer	-
K	true airspeed correction factor	-
$\underline{n}$	unit vector normal to the course	-
p	I.C.A.N. pressure height	mb
q	true airspeed uncorrected	kt
$\underline{q}$	true airspeed vector uncorrected	-
$\underline{s}$	unit vector along the course	-
T_i	indicated air temperature	°C
T	true air temperature	°C
t	time	min

/u

Symbol	Definition	Units
$\underline{u} = K\underline{q}$	the corrected true airspeed vector	-
$U = Kq$	the corrected true airspeed	kt
$\underline{V}$	the ground speed vector	-
V	the ground speed	kt
V_i	the indicated airspeed	kt
V_e	the equivalent airspeed	kt
$\underline{w}$	the wind vector	-
W	the complex wind vector	-
w	the wind speed	kt
w_1	the component of the radiosonde wind along θ	kt
w_2	the component of the radiosonde wind perpendicular to θ	kt
var (x)	the variance of the quantity x	
$\bar{x}$	the mean value of the quantity x	

3. Instruments and their accuracy

3.1. The observations were taken in two ways:-

(1) By visual methods in the pressure cabin of the aircraft.

(2) By photographic methods.

3.2. Visual method

The observer took readings at stated intervals (usually one minute in the Geometrical Flights and three to five minutes in the Comparison Flights) of the following quantities and to the stated accuracy:-

(a)	Time	assumed exact.
(b)	Altitude in feet	probably, when corrected for all known errors, ±100 ft maximum error using a pressure altimeter.
(c)	Heading	assumed exact by standard aircraft master compass, (see para. 1.2.1.).
(d)	Temperature	± 2°C maximum error, measured by standard flat plate thermometer and bridge. (Ref. 1).
(e)	Ground speed	assumed exact, using electronic equipment.
(f)	Drift angle	to be found.
(g)	Airspeed	to be found after correction for existing errors.

/3.3.

3.3. Photographic method: auto-observer readings

A specially designed mounting was made for the following instruments:- clock (time), altimeter (altitude), aircraft master compass independently synchronised (heading), airspeed meter (airspeed) and duplicates of those instruments in the pressure cabin from which drift and ground speed are read. Photographs of the meters were taken at intervals of about 50 seconds (or about six miles of flight) by a camera controlled from the pressure cabin. This camera was fitted with a special heater muff to prevent icing.

3.4. Care was taken to synchronise the clock on the auto-observer with the observer's watch. In general it would be expected that the results from the visual readings would agree with those obtained by the photographic methods as both depend on indications from similar instruments. This is so except when the readings are varying rapidly and the visual readings become a subjective mean over an interval of time whilst the photographic values remain those at a given instant. In this work this effect did not give rise to any difficulties except in the case of the drift meter. In flight, this instrument, which is calibrated in degrees, did not generally show a steady reading but varied slowly and irregularly by 1° to 2° about a mean value. This is probably due to either instrumental hunt or variation in pitch, yaw, etc., not reflected in the compass readings, or possibly a combination of both. The exact cause is unimportant to the text of this paper which is concerned only with the standard error in measurements made up to date using the meter. However, it should be noted that the observer must necessarily take a mean value over some interval (usually about one minute in duration) and that his readings can only be made to 0.25° at best and 0.5° at worst. In all flights readings were taken visually to the nearest 0.5°. In the auto-observer the camera takes an instaneous photograph and so the individual readings may be in error by as much as 2° in the worst possible case: if the error is random this will be reflected in the statistics as a large standard deviation for the error in drift angle (i.e., the standard error of the error in drift angle obtained using the photographic data would be larger than that obtained by visual methods although the means would not differ greatly).

3.5. Recently a modification has been introduced to reduce the instrumental hunt by increasing the servo-damping in the drift unit and preliminary results indicate that the hunt is now about ±0.5° so that further work using the auto-observer is likely to present less variability than that examined up to date.

3.6. In each case, the readings of heading, ground speed and drift were directly available; that of true airspeed was obtained from the indicated temperature, airspeed and height using standard expressions in the form:

$$q = V_e \left\{ \frac{3.53\ (T+273)}{p} \right\}^{\frac{1}{2}} \qquad (1)$$

$$T = T_i - \frac{V_e^2}{100^2} \cdot \frac{a\ 3.53\ (T+273)}{p} \qquad (2)$$

where V_e is the equivalent airspeed obtained by correcting the indicated airspeed V_i for instrumental calibration, compressibility and pressure error.

/4.

4. Geometrical Flights

4.1. Providing the wind can be assumed constant over reasonably short intervals of space and time, measurements of wind and airspeed can be obtained directly from the readings of the ground speed meter by flying various geometrical paths through the air. The simplest of these paths is to fly at constant airspeed on reciprocal compass readings of equal time duration using headings such that the drift is zero. In this case it is assumed that the two legs have equal following and head winds so that the airspeed is equal to the mean of the ground speeds and the wind speed is the difference between the ground speed and the airspeed. These ideas can be elaborated by flying in other patterns (e.g., squares or triangles). In all, six flights of this type were made. The theory of the method is given in Appendix II. Another method which can be used for this type of investigation (although it was developed for another purpose) has been suggested by C.S. Durst (unpublished) and using the same data he has obtained results in good agreement with those quoted below.

4.2. Provided the heading of the aircraft is maintained at constant altitude there is no necessity for it to perform any special turns at the corners of any geometrical flight. The theory merely requires equal times in any two reciprocal directions and constancy of wind. The aircraft therefore climbed to the height at which the pattern was to be flown, and when level, carried out the requisite number of legs (two, three, or four) in the requisite number of directions performing shallow banks at the end of each leg so that the equipment did not become unlocked. The duration of each leg was usually four minutes or about 25 miles of flight.

4.2.1. Flights in a square

Four legs were flown in the form of a square. If the mean airspeed at the j^{th} minute was q_{ij} on the i^{th} leg and the ground speed was V_{ij}, the drift angle α_{ij}, then the formulae (11) and (12) of Appendix II become:

$$\tan\gamma = -\frac{\sum_i \sum_j V_{ij} \sin\alpha_{ij}}{\sum_i \sum_j V_{ij} \cos\alpha_{ij}} \qquad \begin{matrix} i = 1, 2, 3, 4 \\ j = 1, 2, 3, 4 \end{matrix} \qquad (3)$$

$$K = \frac{\sum_i \sum_j V_{ij} \cos(\alpha_{ij} + \gamma)}{\sum_i \sum_j q_{ij}} \qquad (4)$$

4.2.2. Flights in an equilateral triangle

Similar equations can be proved to hold as in 4.2.1. with $i = 1,2,3$ and $j = 1,2,3,4$.

4.2.3. Flights on reciprocal headings

Here, equations (3) and (4) hold with $i = 1$ and 2 and $j = 1,2,3,4$.

/4.3.

4.3. Table I gives values of K and γ calculated for squares, a triangle and reciprocal flights.

Table I

Values of K and γ derived from Geometrical Flights

Date	Height ft	Pattern	Observer		Photographic method		Pilot	Observer
			K	γ deg	K	γ deg		
17.11.55	25,000	Triangle	1.034	-0.23	1.020	-0.29	A	X
	35,000	Triangle	1.028	-0.27	1.026	-0.40	A	X
8.11.55	20,000	Square	1.025	+0.43	1.023	+0.12	B	Y
	40,000	Square	1.027	+0.08	1.025	+0.11	B	Y
9.11.55	10,000	Square	1.024	-0.20	1.015	+0.21	A	Z
	30,000	Square	1.042	+0.21	1.033	+0.31	A	Z
1.11.55	5,000	Reciprocal	1.016	+0.65	1.028	+0.75	B	Y
	10,000	"	1.018	+0.55	1.020	+0.72	B	Y
	20,000	"	1.012	+0.43	1.018	+1.19	B	Y
	25,000	"	1.025	+0.48	1.020	+0.74	B	Y
	30,000	"	1.026	+0.56	1.025	+0.75	B	Y
7.11.55	30,000	"	1.025	+0.58	1.023	+0.10	B	Y
	35,000	"	1.037	+0.39	1.039	+0.25	B	Y
	40,000	"	1.033	+0.50	1.027	+0.60	B	Y
	45,000	"	1.016	+0.82	1.014	+1.28	B	Y
21.10.55	15,000	"	1.025	+0.75			B	X
	20,000	"	1.029	+0.82			B	X
	30,000	"	1.027	+0.82			B	X
	40,000	"	1.031	+0.33			B	X

Mean value of K from:-		Variance
visual readings	1.026	$0.54 \cdot 10^{-4}$
photographic readings	1.024	$0.39 \cdot 10^{-4}$
all readings	1.025	$0.62 \cdot 10^{-4}$

Mean value of γ from:-		Variance (rad^2)
visual readings	+0.41 deg.	$0.35 \cdot 10^{-4}$
photographic readings	+0.43 "	$0.68 \cdot 10^{-4}$
all readings	+0.42 "	$0.49 \cdot 10^{-4}$ (0.40 deg^2)

5. Comparison Flights

5.1. As already mentioned, these were comparisons between mean winds determined by radiosonde and winds determined by the aircraft. The opportunity was taken at the same time to make similar comparisons of frost points and temperatures, but only the comparisons of winds will be considered here.

/5.2.

5.2. Since the balloon ascends at 1,200 ft per minute and also changes its plan position irregularly due to changes of wind it was evidently useless to attempt synchronous readings. The procedure adopted was to time the aircraft's flight so that, when the balloon left the ground, the aircraft observer had already taken several measurements of frost point at standard levels. These standard levels are at 5,000 ft intervals from 5,000 ft to 30,000 ft and then at 2,000 ft intervals from 32,000 ft to 48,000 ft with one extra reading at 35,000 ft. Before and after the frost point was measured, readings were taken of all the other parameters:- temperature, airspeed, ground speed, heading, drift and height. The flight usually ended about an hour after the balloon had reached 50,000 ft. The auto-observer was again used to duplicate the readings and there were many more readings available from this instrument than the two taken visually at any level.

5.3. Theory

Consider components of the wind, airspeed and ground speed along and perpendicular to the heading: let w_1 be the radiosonde wind component along the heading and w_2 be the radiosonde wind component perpendicular to the heading, so that:-

$$V \cos(\alpha + \gamma) = U + w_1 \qquad (5)$$

$$V \sin(\alpha + \gamma) = w_2 . \qquad (6)$$

Equation (6) can be solved for γ since all the other quantities are known.

$$\gamma = -\alpha + \sin^{-1}\frac{w_2}{V} . \qquad (7)$$

Using this value of γ, put $U = K q$ in equation (5) and solve for

$$K = \frac{V \cos(\alpha + \gamma) - w_1}{q} . \qquad (8)$$

5.4. Analysis

The radiosonde wind is a mean over an interval of one minute and it was resolved into north and east components using the raw data. These components were taken at the mean height of the layer through which the balloon travelled during the time interval. Graphs of north and east components against geometric height were plotted and the radiosonde temperatures were used to convert these heights into I.C.A.N. pressure heights. The corrected I.C.A.N. height of each observation taken in the aircraft is known and so a wind can be estimated in the form of north and east components at the height of each aircraft observation. These components were resolved along and perpendicular to the indicated heading of the aircraft giving w_1 and w_2. Formulae (7) and (8) were then used to derive values of K and γ for each observation.

5.5. Values of K and γ were plotted against height, indicated airspeed, drift angle and against each other. No relationship was found. Table II gives mean values of K and γ obtained on flights by both the visual readings and the photographic readings together with the variance about these means. Separate overall means and variance are listed for visual and photographic values irrespective of date and the gross means and variances (visual plus photographic readings) are also listed. The total number of values is 432.

/Table II

TABLE II

Values of K and γ derived from Comparison Flights

Date	Pilot	Readings taken visually					Readings taken photographically					Observer
		No. of observations	Mean K	Variance of K x 10^4	Mean γ deg	Variance of γ deg^2	No. of observations	Mean K	Variance of K x 10^4	Mean γ deg	Variance of γ deg^2	
28.10.55	A	19	1.027	11.7	+0.50	1.50	80	1.026	5.2	+0.71	1.64	X
31.10.55	B	19	1.033	0.1	+1.20	0.79	41	1.034	4.0	+0.86	0.58	X
1.11.55	A	17	1.017	5.6	+0.87	0.46	55	1.019	4.0	+1.11	0.79	X
8.11.55	A	19	1.030	3.4	+0.70	1.20	66	1.025	2.3	+0.71	1.15	X
22.11.55	B	17	1.025	6.5	+0.60	0.92	21	1.012	8.1	+0.43	1.11	X
28.11.55	C	15	1.021	4.8	+0.83	0.80	50	1.025	1.4	+0.72	1.81	X
29.11.55	B	13	1.017	5.0	+0.40	1.96	-	-	-	-	-	X
Mean		TOTAL 119	1.025	5.9	+0.74	1.18	TOTAL 313	1.024	8.2	+0.71	1.30	
Gross Mean		TOTAL 432	1.025	7.5	+0.72	1.25 = 3.84 x 10^{-4} radians^2						
Mean value of means for each flight $\overline{K}$ = 1.025 Variance of $\overline{K}$ = 0.39 . 10^{-4} $\overline{\gamma}$ = +0.72 Variance of $\overline{\gamma}$ = 0.23 . 10^{-4} radians^2												

6. Discussion

6.1. It is very difficult to separate the sources of error and apportion the magnitudes in this type of work and so the first step in the analysis was to construct Table III which shows mean values of K and γ for each height obtained on all flights but separated into four classes as shown in the headings. Means of each column and corresponding variances are also given.

/Table III

Table III

Variations of K and γ with height

(Number of observations in brackets)

Height ft	Geometrical Flights				Comparison Flights			
	Visual K	Photo K	Visual γ deg	Photo γ deg	Visual K	Photo K	Visual γ deg	Photo γ deg
5,000	1.016(1)	1.028(1)	+0.65	+0.75	1.026(5)	1.016(4)	+0.43	+0.91
7,000	-	-	-	-	1.049(1)	1.016(1)	+0.73	+0.30
10,000	1.021(2)	1.017(2)	+0.17	+0.46	1.029(7)	1.021(6)	+0.64	+1.09
15,000	1.025(1)	-	+0.75	-	1.017(7)	1.020(6)	+0.53	+0.88
20,000	1.022(3)	1.020(2)	+0.56	+0.66	1.024(7)	1.018(6)	+0.66	+0.70
25,000	1.029(2)	1.020(2)	+0.13	+0.22	1.037(6)	1.030(6)	+0.55	+0.20
30,000	1.030(4)	1.027(3)	+0.54	+0.39	1.031(7)	1.029(5)	+0.79	+0.17
34,000	-	-	-	-	1.005(1)	1.067(1)	+1.95	+1.49
35,000	1.032(2)	1.032(2)	+0.06	-0.07	1.040(5)	1.013(1)	+0.84	+1.45
36,000	-	-	-	-	1.022(2)	1.027(4)	+0.14	+0.76
38,000	-	-	-	-	-	1.012(1)	-	-1.80
40,000	1.030(3)	1.026(2)	+0.31	+0.36	1.013(6)	1.012(4)	+0.65	+0.65
42,000	-	-	-	-	-	1.017(2)	-	+1.70
43,000	-	-	-	-	-	1.030(1)	-	+1.89
44,000	-	-	-	-	-	1.037(2)	-	+1.52
45,000	1.016(1)	1.014(1)	+0.82	+1.28	-	-	-	-
46,000	-	-	-	-	1.019(7)	1.028(5)	+0.63	+1.01
48,000	-	-	-	-	1.018(7)	1.032(5)	+0.81	+0.57
Mean	1.025(9)	1.023(8)	+0.44	+0.51	1.025(13)	1.025(17)	+0.72	+0.78
Variance x 10^4	0.336	0.332	0.216 (rad^2)	0.43 (rad^2)	1.016	1.74	0.48 (rad^2)	2.11 (rad^2)

At first sight there would appear to be a correlation of K with height but the mean values of γ are so variable that no correlation with height is apparent. In order to find the correlation of K with height (expressed in thousands of feet), the Geometrical Flight data were used and gave the following results:-

(a) Visual readings

Correlation coefficient +0.39
Regression line $K - 1.025 = (1.88).10^{-4} \times (\text{height} - 25)$.

(b) Photographic readings

Correlation coefficient -0.03
Regression line $K - 1.023 = (1.44).10^{-5} \times (26.25 - \text{height})$.

The probable error of the value of correlation coefficient was 0.2 in each case, and it is considered that no significant variation with height has been shown by the above analysis. The correlation of Comparison Flight data with height would be heavily masked by errors to be discussed later. It was decided therefore to find the best values of K and γ without regard to variation with height for both types of flight.

/6.2.

6.2. In the Geometrical Flights, the values of K and γ obtained at any level on a given day are necessarily mean values and any such values can be considered as a separate flight in the analysis. In the Comparison Flights, however, the values of K and γ on each flight are found from isolated readings and so, besides the separation into heights, it is possible to find means for a flight irrespective of height. There are, therefore, several means involved in the analysis and these will be discussed before being applied to the data available.

(a) Geometrical Flights

(1) Mean K and γ can be found for each height (Table III) and for all days and finally means of these for all heights can be found.

(2) Mean K and γ can be found for all the determination in Table I. The means are differently weighted from those in para. (a)(1) in a manner which depends on the number of evaluations at each height.

(3) Variances can be found for both K and γ about the overall mean values in para. (1) and (2) above.

(4) Since there are two separate sets of observations - visual and photographic - these can be combined before operation (a)(1) is carried out and this will give values for mean K and mean γ. This smoothing of the data, however, will introduce observational errors which cannot be assessed, and for this reason it was considered that this process is not justified.

(5) Student's t-test can be used when operations described in para. (2) and (3) have been performed separately on the visual and photographic values of K and γ to discover whether they are significantly different.

(b) Comparison Flights

(1) Mean values can be found for each day without regard to height and variances about these means can be computed. (Table II).

(2) Mean values can be found for each height on each day and then the mean value for all days at each height can be derived. Variance can be computed about each of these means. (Table III).

(3) Student's t-test can be used to examine the statistical difference between the values of K and γ derived from visual readings and those derived from photographic readings in both para. (b)(1) and (b)(2).

6.3. Since it seems very doubtful from para. 6.1. whether K and γ vary with height, the meaning process described in para. 6.2.(a)(2) was used for both sets of observational data (visual and photographic) on the Geometrical Flights. Student's t-test showed that at the 5 per cent level of significance there was no difference between these two observational methods. Variances of K and γ corresponding to these means were also found. These variances are in part due to (a) time and space variations of the wind during the period of measurement and in part due to (b) instrumental variations. If (a) could be estimated (b) would be determinate. In order to do this, the mean values of K and γ found above were used to find individual winds $\underline{w}_i$ and mean winds $\underline{\bar{w}}$ for each flight. The root mean square differences between each individual wind and these means were found and summed in the form $\sum \frac{1}{n} (\underline{w}_i - \underline{\bar{w}})^2$ Results are given in Table IV.

/Results

Table IV

Geometrical Flights

Comparison of root mean square differences with probable variation of wind

Height ft	Date	Track	Root mean square diff. kt	Mean wind kt	Variability of wind (Durst) kt	
					Distance	Time
20,000	8.11.55	Square	6.33	33.6	12	2.5
40,000		"	6.36	39.5	11	3.5
10,000	9.11.55	"	8.76	34.5	7.5	2
30,000		"	13.01	31.1	15	6
25,000	17.11.55	Triangle	8.31	51.3	12.5	4
35,000		"	8.32	72.4	12	4
30,000	7.11.55	Reciprocal	5.72	68.1	12	4
35,000		"	6.35	61.8	11	4
40,000		"	4.85	58.9	9	3.5
45,000		"	6.36	33.9	7.5	2.5
5,000	1.11.55	"	3.14	11.1	6	2
10,000		"	4.12	4.7	7	2
20,000		"	5.48	14.4	10	2.5
25,000		"	3.24	21.7	11.5	3.5
30,000		"	3.96	24.8	12	4
15,000	21.10.55	"	5.95	30.9	7.5	2
20,000		"	5.60	17.9	10	2.5
30,000		"	8.63	33.4	12	4
40,000		"	7.13	4.2	9	3.5

Also included in this table are estimates of the distance and time variation to be expected from Fig. 5 of Geophysical Memoir 93(2) using representative flight durations and distances. It can be seen that the root mean square differences found above are in all cases less than the expected time and distance variations. Hence it is impossible to separate the two sources of variance. However, it is evident that the standard errors of wind measurements cannot exceed, and may be considerably less than, the values indicated in column 4 of Table IV.

An estimate of the standard error of a wind reading may be given using equation (9) of Appendix 1 with the results in Table I, i.e., $\mathrm{var}(K) = 0.54.10^{-4}$, $\mathrm{var}(\gamma) = 0.35.10^{-4}$ rad^2 for visual readings and $\mathrm{var}(K) = 0.39.10^{-4}$, $\mathrm{var}(\gamma) = 0.68.10^{-4}$ rad^2 for photographic readings. With $K = 1.025$ and $\gamma = +0.5$, typical values using the data of the example in Appendix I, shows, for visual readings, the standard error is 4.5 kt, and for photographic readings 5.0 kt.

/6.4.

6.4. The Comparison Flights were analysed in both of the ways outlined in para. 6.2.(b). The results of para. 6.2.(b)(1) are shown in Table II and those of 6.2.(b)(2) in Table III. Non-weighted means were used in all cases. It will be seen from these tables that, for any particular day or any particular height, the mean K is 1.025 by both methods of analysis and for the two different types of observation and that the mean γ is near $+0.75^{\circ}$. An attempt was also made to separate space and time variations from instrumental effects. The effects of space and time variation involved here are much larger than in the Geometrical Flights since the compared values may be at a time difference of one hour and 50 miles apart. Table V(a) shows the expected vector variation in time and space of the wind (based on Geophysical Memoir No. 93)(2). For comparison purposes, the values of var(K) and var(γ) were taken from Table II and inserted into equation (9) of Appendix I as in 6.3.

Table V(a)

Wind Errors in Comparison Flights

Height ft	Variability of wind (Durst) kt 50 mile distance	1 hour time
5,000	7.0	4.0
10,000	7.5	4.5
15,000	8.5	5.0
20,000	12.0	6.5
25,000	14.0	8.0
30,000	15.0	8.5
35,000	13.0	8.0
40,000	10.5	6.5
45,000	8.0	5.0
50,000	6.5	4.0

Table V(b)

Date	Wind error (kt) Visual	Photographic
28.10.55	17.36	14.42
31.10.55	7.86	10.40
1.11.55	11.16	11.15
8.11.55	12.07	11.15
22.11.55	13.19	14.63
28.11.55	11.74	12.73
29.11.55	15.14	

The specimen errors produced, shown in Table V(b), are of the same magnitude as the vector variation in Table V(a) or slightly larger. In view of the failure in 6.3. above to produce any adequate discrimination of the two types of errors by using the data of Geophysical Memoir No. 93(2) in this way it was not considered worthwhile to attempt to assign quantitative values to the instrumental error in this case.

6.5. From the various values of standard variance shown in Tables I, II and III it is difficult to choose the one which should be used finally in estimating the error of the wind. The most useful figures to apply in practice to visual observations are var$(\gamma) = 0.4.10^{-4}$ rad^2 and var$(K) = 0.5.10^{-4}$ these figures being reasonable estimates allowing for the variation in experimental methods and averaging processes. As discussed in para. 6.3. and 6.4. above the cause of the scatter is not known although it appears likely that the major cause is not instrumental.

/6.6.

6.6. Whatever the averaging process employed in the analysis of the visual observations the results in the mean suggest a value of $K = 1.025$ and this will, in general, be the nearest practical estimate of the airspeed correction factor. Table VI gives suggested corrections for the speed ranges shown which should be added to the true airspeed.

Table VI

Corrections to true airspeed

True airspeed (kt)	Correction (kt)
280 to 299	+7
300 to 339	+8
340 to 379	+9
380 to 419	+10
420 to 459	+11
460 to 499	+12

This rough correction is not suggested for accurate wind finding in which the factor 1.025 should be applied directly to the data, but can be of value to the navigator in routine work using this particular installation.

6.7. The value of γ is subject to various calibration errors but it is suggested that $+0.5^\circ$ is a reasonably practical quantity which could be added to all drift readings when starboard drifts are counted positive.

6.8. The error is shown to be 11 kt for any single wind reading in Appendix I when no special corrections are applied. Using $K = 1.025$ $\gamma = +0.5^\circ$ and $\mathrm{var}(K) = 0.5.10^{-4}$ and $\mathrm{var}(\gamma) = 0.4.10^{-4}$ rad^2 the error of a wind determination is shown to be 4.5 kt approx. and in a continuing series of observations it is probably much less than this. More recent work on long flights using the above values of K and γ with readings at intervals of one minute, suggests that errors in continuing sequences may be as small as one knot when used comparatively, although the absolute value of the wind vector may have a considerably larger error.

6.9. For the photographic readings, although the mean values of K and γ can be applied as for the visual readings, it is desirable to assume larger variances since the errors are somewhat larger due to the fact that the drift meter varies in a random fashion. It seems from the tables that values $\mathrm{var}(K) = 0.4.10^{-4}$, $\mathrm{var}(\gamma) = 0.7.10^{-4}$ rad^2 should be used. This gives the error of a wind reading taken photographically to be 5 kt. In the case of photographic readings, however, this is the resolution of the system since in any continuing series of observations it is unlikely that greater accuracy can be obtained because of the variations on the drift meter.

7. Conclusions

7.1. The equipment giving ground speed and drift has now been in regular use at the Meteorological Research Flight for about 18 months; during this period its serviceability has been extremely good but there are some limitations. Over calm seas and in steep banks or turns this equipment sometimes gives erroneous values since the basic radio principle involved requires straight and level flight with good surface reflections. To the observer this "unlocking" can be detected by indicators on the display panel but some selection of the photographic observations is occasionally necessary. This limitation has caused little trouble in routine meteorological work.

7.2. When a fault occurs on the apparatus and the maintenance has been carried out, there sometimes appears to be a change in calibration and calibration checks of the type described in section 4 are necessary before any large scale investigation is undertaken.

/7.3.

7.3. The reduction of the data obtained is straightforward but laborious particularly when auto-observer techniques are used. There is a requirement for some form of automatic computer which will give direct presentation of the wind data in flight.

7.4. In the comparative type of work for which this instrumental system will be used, e.g., jet streams, the errors involved will be fairly small and can be estimated roughly by the methods shown in Appendix I. In work of a more absolute nature, e.g., variation of wind in time and space, some further improvement is desirable but this improvement should be attempted in the pressure indicating instruments not in the electronic system.

8. Summary

An extremely useful equipment is now available for measurement of wind from aircraft for meteorological investigations.

It is suggested that (with the present installations on the aircraft of the Meteorological Research Flight) if the true airspeed is multiplied by a factor of 1.025 and +0.5° is added to all drift angles - starboard drift angles being positive - the error in a wind measurement using this system will be less than 5 kt.

9. Bibliography

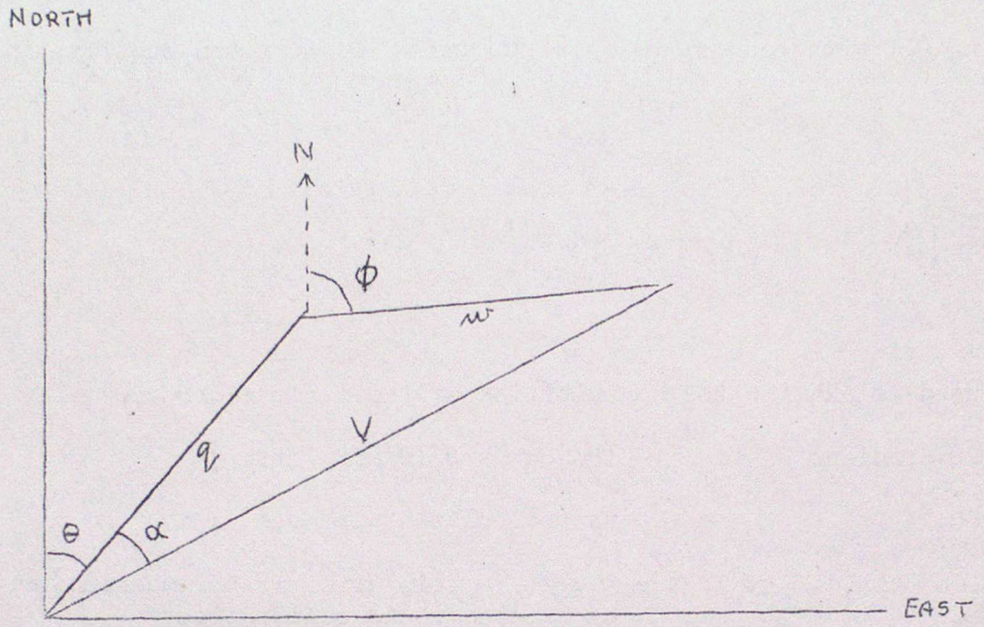
1. Durbin, W.G. 1955 Meteorological Research Papers, Air Ministry, London, No. 930.

2. Durst, C.S. 1955 Geophysical Memoirs No. 93, Air Ministry, London.

APPENDIX I

The calculation of the wind from ground speed, airspeed, drift angle and heading and the effect of errors in these quantities

A.1.1.



Referring to Fig.1, in vector notation:-

$$\text{the wind} \qquad \underline{w} = \underline{V} - \underline{q} \tag{1}$$

or in cartesian co-ordinates

$$w \cos \phi = V \cos (\theta + \alpha) - q \cos \theta \tag{2}$$

$$w \sin \phi = V \sin (\theta + \alpha) - q \sin \theta \tag{3}$$

w and ϕ can be calculated using measured independent values of V, q, θ and α from (2) and (3).

A.1.2. Rewriting (2) and (3) in complex form

$$W = w e^{i\phi} = V \exp i (\theta + \alpha) - q \exp i \theta . \tag{4}$$

/If

If errors ΔV, Δq, $\Delta \theta$ and $\Delta \alpha$ occur in the respective measured values, the total mean square vector error in W is given by

$$|\Delta W|^2 = \left|\frac{\partial W}{\partial V}\right|^2 |\Delta V|^2 + \left|\frac{\partial W}{\partial q}\right|^2 |\Delta q|^2 + \left|\frac{\partial W}{\partial \theta}\right|^2 |\Delta \theta|^2 + \left|\frac{\partial W}{\partial \alpha}\right|^2 |\Delta \alpha|^2 \qquad (5)$$

Differentiating (4) with respect to V, q, θ and α in turn and substituting in (5)

$$|\Delta W|^2 = |\Delta V|^2 + |\Delta q|^2 + w^2 |\Delta \theta|^2 + V^2 |\Delta \alpha|^2 \qquad (6)$$

which can be used to examine the contribution of the various errors. If

$\Delta \theta \simeq$ 0.01 radians - the compass reading error;

w = 100 kt - the calculated wind speed;

$\Delta V \simeq$ 0.003 V = 1.5 kt if V = 500 kt. This error is the maximum error by an electronic calibration;

$\Delta q \simeq$ 0.025 q = 10 kt if q = 400 kt - determined experimentally in the text above;

$\Delta \alpha \simeq$ 0.008 radians - determined experimentally in the text above;

then

$$|\Delta W|^2 = 1 + 2.25 + 100 + 16 = 119.25.$$

So $|\Delta W|$ = 11 kt when no special corrections are made. If those due to Δq and $\Delta \alpha$ could be eliminated by calibration, $|\Delta W|$ could be reduced to about 2 kt in the most favourable circumstances. In practice this will not be achieved due to reading and random errors and these are considered further below.

A.2.1. The object of the calibrating flights was to find mean corrections K to q and γ to α, where

$$u = Kq \quad \text{and} \quad \beta = \gamma + \alpha \qquad (7)$$

where u is the corrected airspeed and β the corrected drift angle, if K and γ can be found exactly $\Delta u = 0 = \Delta \beta$ and the equation (6) reduces to

$$|\Delta W|^2 = |\Delta V|^2 + w^2 |\Delta \theta|^2$$

A.2.2. If the determination of K and γ involves errors ΔK and $\Delta \gamma$

$$\Delta u = K \Delta q + q \Delta K \quad \text{and} \quad \Delta \beta = \Delta \gamma + \Delta \alpha$$

/and

and $\Delta q = 0 = \Delta\alpha$ since these quantities have already been corrected by K and γ. Hence $\Delta u = q\,\Delta K$ and $\Delta\beta = \Delta\gamma$ and (6) becomes

$$|\Delta W|^2 = |\Delta V|^2 + w^2|\Delta\theta|^2 + q^2|\Delta K|^2 + V^2|\Delta\gamma|^2 \qquad (8)$$

where w is the value of the wind using the corrected values for airspeed and drift angle.

A.2.3. Since $(\Delta K)^2$ and $(\Delta\gamma)^2$ were examined as variances about the mean and $(\Delta V)^2$ and $(\Delta\theta)^2$ are known as maximum errors, using variances (8) becomes

$$\text{var}\,(W) \simeq \left|\frac{\Delta V}{3}\right|^2 + w^2\left|\frac{\Delta\theta}{3}\right|^2 + q^2\,\text{var}(K) + V^2\,\text{var}(\gamma). \qquad (9)$$

A.3. It would not be worthwhile at the present stage to attempt to carry this analysis further since variations of wind with time and space have been unavoidable in the experimental procedure used so far.

APPENDIX II

Theory of the geometrical flight calculations

A.2.1. Consider a path in the air whose unit normal vector is $\underline{n}$ and whose unit tangent vector is $\underline{s}$. The wind equation is

$$\underline{V} = \underline{U} + \underline{w}.$$

In time dt the distance moved along the path is

$$\underline{U} \cdot \underline{s}\, dt = \underline{V} \cdot \underline{s}\, dt - \underline{w} \cdot \underline{s}\, dt \qquad (1)$$

and the distance moved perpendicular to the path is

$$\underline{U} \cdot \underline{n}\, dt = \underline{V} \cdot \underline{n}\, dt - \underline{w} \cdot \underline{n}\, dt \qquad (2)$$

Now $\underline{qs}$ is the uncorrected airspeed vector. Put $\underline{U} = K\,\underline{qs}$ where K is some correcting factor.

$$\therefore \quad \underline{U} \cdot \underline{n} = Kq\,\underline{s} \cdot \underline{n} = 0$$

and equations (1) and (2) become

$$\underline{V} \cdot \underline{s}\, dt = (Kq + \underline{w} \cdot \underline{s})\, dt \qquad (4)$$

$$\underline{V} \cdot \underline{n}\, dt = \underline{w} \cdot \underline{n}\, dt \qquad (5)$$

and if

$$\int_0^t \underline{w} \cdot \underline{n}\, dt = 0 = \int_0^t \underline{w} \cdot \underline{s}\, dt \qquad (6)$$

and $\underline{w}$ is constant, (4) and (5) on integration with respect to t

$$\int_0^t V \cdot s\, dt = \int_0^t Kq\, dt \qquad (7)$$

$$\int_0^t \underline{V} \cdot \underline{n}\, dt = 0. \qquad (8)$$

Reverting to Cartesian co-ordinates, equation (6) means that the flight plan has to be such that the time spent flying an easterly component must be balanced by an equal time spent flying a westerly component and similarly with the north-south directions, e.g., reciprocal headings, squares and equilateral triangles with equal times of flight on each leg. If constant airspeed is maintained, any closed circuit which returns to the original air position satisfies the conditions.

/A. 2. 2.

A.2.2. If there is an error γ in the angle of drift α and V is the modulus ground speed

$$\underline{V} \cdot \underline{n} = V \sin(\alpha + \gamma) = V \sin\alpha \cos\gamma + V \cos\alpha \sin\gamma \quad (9)$$

$$\underline{V} \cdot \underline{s} = V \cos(\alpha + \gamma) \quad (10)$$

using the expanded form of (9) and allowing γ to be independent of time, equation (8) becomes:

$$\int_0^t V \sin\alpha \cos\gamma \, dt = - \int_0^t V \cos\alpha \sin\gamma \, dt$$

$$\therefore \quad \tan\gamma = - \frac{\int_0^t V \sin\alpha \, dt}{\int_0^t V \cos\alpha \, dt} \quad (11)$$

This enables a value of γ to be calculated for each complete geometrical pattern and when this value is known and equation (10) is substituted in equation (7)

$$\int_0^t Kq \, dt = \int_0^t V \cos(\alpha + \gamma) \, dt$$

and if K is independent of t

$$K = \frac{\int_0^t V \cos(\alpha + \gamma) \, dt}{\int_0^t q \, dt} \quad (12)$$